

MATH 42-NUMBER THEORY
PROBLEM SET #8
DUE TUESDAY, APRIL 19, 2011

1. Compute

$$\sum_{k=1}^5 \left\lfloor \frac{5 \cdot k}{11} \right\rfloor.$$

Use this to determine whether 5 is a square mod 11 or not.

2. Compute

$$\sum_{k=1}^6 \left\lfloor \frac{5 \cdot k}{13} \right\rfloor.$$

Use this to determine whether 5 is a square mod 13 or not.

3. Graph the line from (0,0) to (11,5). How many lattice points (that is, points (a,b) where a and b are in $\mathbb{Z}$) are there below this line and above the x -axis with first coordinate 1? With first coordinate 2? 3? 4? 5? How does this relate to the sum from problem 1?
4. Graph the line from (0,0) to (13,5). How many lattice points are there below this line and above the x -axis with first coordinate 1? With first coordinate 2? 3? 4? 5? 6? How does this relate to the sum from problem 2?
5. Prove that if p and q are distinct odd primes, then

$$\sum_{k=1}^{\frac{p-1}{2}} \left\lfloor \frac{kq}{p} \right\rfloor + \sum_{\ell=1}^{\frac{q-1}{2}} \left\lfloor \frac{\ell p}{q} \right\rfloor = \frac{p-1}{2} \cdot \frac{q-1}{2}.$$